



**IN THE NAME OF ALLAH,
MOST GRACIOUS, MOST MERCIFUL**

DIFFERENTIAL EQUATIONS

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Preface

Ordinary Differential equations are a subject that is accorded a great deal of interest, and it is in fact a subject that plays a crucial role in modeling physical and scientific phenomena that occur daily.

The aim of this book is to explain and discuss elementary ordinary differential equations. Most of engineering and science students have at least one course in ordinary differential equations in their whole cursus. This book is intended for students of engineering and science undergraduate students. The book provides students with a rich background on ordinary differential equations on theory and applications. The reader is presumed to have a good background in calculus of one variable and in linear algebra and matrices.

We have included a large number of examples and review exercises in each section of each chapter. The exercises are generally classified by order of difficulty and are mostly applications of the material covered in the text.

Students who study seriously most of the examples given in each section and try to solve most of the exercises will surely develop an appreciation for the importance of ordinary differential equations and will also acquire great skills in mathematical modeling of differential equations.

Because of their easiness, we started in chapter two by various types of first order linear and nonlinear differential equations such as separable equations, exact and non exact equations (integrating factor), equations with linear coefficients, Bernoulli equations. The student will learn many methods for solving different types of first order differential equations. The main theoretical result in this chapter is the theorem of existence and uniqueness of solution of an initial value problem. Chapter three helps students to model a certain physical phenomena by a first order differential equation. The main applications are those related to Newton's law of cooling, growth and decay (populations, radioactive materials) and orthogonal trajectories. Higher order differential equations are treated in chapter four. Students will master and learn adequate methods for solving higher order equations with constant coefficients and those with variable coefficients. Variation of parameters method will generally be applied for both kinds of differential equations, but undetermined coefficients method will be restricted to only equations with constant coefficients. A more powerful method called power series method, can be applied to equations of variable coefficients is treated in chapter five. Acquiring more skills on differential equations, the student can deal with ordinary differential equation systems with constant coefficients which are the content of chapter six. In chapter seven, we treat periodic functions which occur and arise in many physical and engineering problems such as in conduction of heat

and mechanical vibrations. It is useful to express these functions in terms of Fourier series. Most of functions that engineering students deal with are those which occur in applied mathematics and can be expressed as Fourier series. The last chapter is devoted to Fourier integrals. Using this tool, students can evaluate and compute definite integrals that cannot be computed by using traditional methods.

We are grateful to some of our colleagues and the students in our classes for pointing out mistakes in the first and second drafts of this book and suggesting a number of improvements.

We would like to greatly thank our dear Colleague Professor Mongi Blel for his considerable and substantial efforts in correcting and improving the Tex version of the book. We are also grateful to some of our other colleagues and the students in our classes for pointing out mistakes in the first and second drafts of this book and suggesting a number of improvements.

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